VIDYA BHAWAN BALIKA VIDYA PITH

शक्तिउत्थानआश्रमलखीसरायबिहार

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1. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

Solution:

Given, a spherical ball of salt

Then, the volume of ball V = 4/3 πr^3 where r = radius of the ball

Now, according to the question we have

dV/dt ∝ S, where S = surface area of the ball

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) \propto 4\pi r^2 \qquad [\because \quad S = 4\pi r^2]$$

$$\frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} \propto 4\pi r^2$$

$$4\pi r^2 \cdot \frac{dr}{dt} = K \cdot 4\pi r^2 \qquad (K = \text{Constant of proportionality})$$

$$\frac{dr}{dt} = K \cdot \frac{4\pi r^2}{4\pi r^2}$$

$$\frac{dr}{dt} = K \cdot 1 = K$$

Therefore, the radius of the ball is decreasing at constant rate.

2. If the area of a circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.

Solution:

We know that the area of circle, A = πr^2, where r = radius of the circle

And, perimeter = $2\pi r$

According to the question, we have

$$\frac{dA}{dt} = K, \text{ where } K = \text{constant}$$

$$\Rightarrow \frac{d}{dt}(\pi r^2) = K \quad \Rightarrow \quad \pi \cdot 2r \cdot \frac{dr}{dt} = K$$

$$\text{So, } \frac{dr}{dt} = \frac{K}{2\pi r}$$

Now, perimeter $c = 2\pi r$

Differentiating w.r.t t, we get

$$\frac{dc}{dt} = \frac{d}{dt}(2\pi r) \quad \Rightarrow \quad \frac{dc}{dt} = 2\pi \cdot \frac{dr}{dt}$$

$$\frac{dc}{dt} = 2\pi \cdot \frac{K}{2\pi r} = \frac{K}{r} \qquad \text{[From (1)]}$$

$$\frac{dc}{dt} \propto \frac{1}{r}$$

Therefore, it's seen that the perimeter of the circle varies inversely as the radius of the circle.

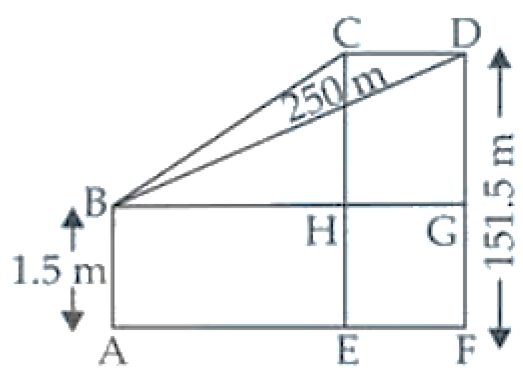
3. A kite is moving horizontally at a height of 151.5 meters. If the speed of kite is

10 m/s, how fast is the string being let out; when the kite is 250 m away from

the boy who is flying the kite? The height of boy is 1.5 m.

Solution:

Given,



Height of the kite(h) = 151.5 m

Speed of the kite(V) = 10 m/s

Let FD be the height of the kite and AB be the height of the kite and AB be the height of the boy.

Now, let AF = x m

So, BG = AF = x

And, dx/dt = 10 m/s

From the figure, it's seen that

GD = DF – GF = DF – AB

= (151.5 – 1.5) m = 150 m [As AB = GF]

Now, in Δ BDG

$BG^2 + GD^2 = BD^2$ (By Pythagoras Theorem)

$x^2 + (150)^2 = (250)^2$

$x^2 + 22500 = 62500$

$x^2 = 62500 - 22500 = 40000$

x = 200 m

Let initially the length of the string be y m

So, in Δ BDG

$BG^2 + GD^2 = BD^2$

$x^2 + (150)^2 = y^2$

Differentiating both sides w.r.t., t, we have

$$2x \cdot \frac{dx}{dt} + 0 = 2y \cdot \frac{dy}{dt} \qquad \left[\because \frac{dx}{dt} = 10 \text{ m/s}\right]$$

$$2 \times 200 \times 10 = 2 \times 250 \times \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{2 \times 200 \times 10}{2 \times 250} = 8 \text{ m/s}$$

Therefore, the rate of change of the length of the string is 8 m/s.